Topic: First Order Linear Recurrence

Time: 45 mins Marks: /45 marks

Calculator Assumed

Question One: [3, 3, 3: 9 marks]

Complete the tables below for the first five terms of each sequence, defined by each recursive rule.

a) $T_n = T_{n-1} + 2n, T_1 = 0$

n	1	2	3	4	5	30
T_n						

b) $a_{n+1} = \frac{a_n}{n+1}$ $, a_1 = 12$

n	1	2	3	4	5	10
a_{n+1}						

c) $U_n = U_{n-1} + U_{n-2}, U_0 = 1\ U_1 = 2$

n	1	2	3	4	5	12
U_n						

Question Two: [2, 4, 2, 2: 10 marks]

Angelena has built a new school. When the school first opens she has 230 students across the school with the population of the school set to increase by 24% each year.

a) Assuming no students leave the school, state the recursive rule to describe the number of students at the end of each year.

For the first two years no students leave the school for other schools and no students are old enough to graduate yet. However, after the initial first two years, 50 students leave the school at the end of each year.

b) Calculate the number of students at the end of the 4^{th} year assuming the population continues to grow at 24% each year.

c) Explain why Angelena would not be worried about losing 50 students per year but would be very concerned if she was losing 100 students per year.

9 years after Angelena opened her new school, Brad opened an even better school very nearby and Angelena's students begin flocking to Brad's new school. Her student population no longer increases but reduces by 24% each year.

d) Describe what happens to Angelana's school over the next few years.

Question Three: [2, 2, 2, 2: 8 marks]

Consider the following sequence:

Term	1	2	3	4	5	n
T_n	1	1+2	1+2+3	1+2+3+4	1+2+3+4+5	$\frac{n(n+1)}{2}$

a) Give the recurrence formula to calculate the n^{th} term.

b) Show that the n^{th} term can be calculated using the formula $\frac{n(n+1)}{2}$.

c) Calculate the 9th term and 10th term.

d) Complete the following table where S_n is the sum of successive terms.

n	1	2	3	4	5
S_n					

Question Four: [2, 2, 2, 5: 11 marks]

Cole buys an antique piano for $2 100. The seller tells Cole that the piano is an investment as its value is certain to increase by 5% each year.

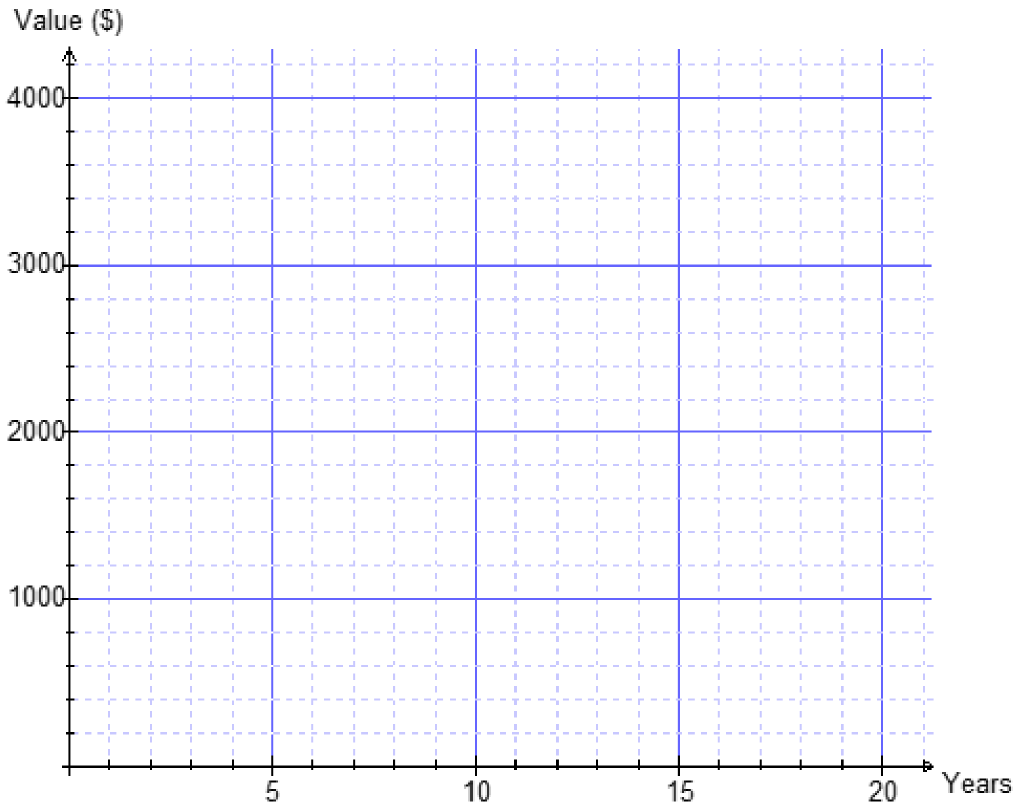
a) Given that Cole purchased the piano for current market value, what will the piano be valued at in 12 years' time?

b) How long will it take before the piano's value is at least $8 000?

Ty is doing some investing and buys $4 050 worth of shares of a relatively new company. Shortly after his purchase the company begins to run into some serious trouble and the value of Ty's investment falls by $100 each year.

b) Give the recurrence formula for the value of Ty's shares each year.

c) Sketch the graph of the value each man's purchase given the patterns described continue. Label the key features and the point at which the value of the piano and shares are equal.



Question Five: [3, 2, 2: 7 marks]

In an arithmetic sequence S_n is the sum of successive terms. Consider the arithmetic sequence where, $S_1 = 8, S_2 = 23, S_3 = 45, S_4 = 103$.

a) Calculate the first three terms of the sequence.

b) Calculate the 18th term of the sequence.

c) Calculate the sum of the first 12 terms.

education equals

Topic: First Order Linear Recurrence
SOLUTIONS

Time: 45 mins Marks: /45 marks

Calculator Assumed

Question One: [3, 3, 3: 9 marks]

Complete the tables below for the first five terms of each sequence, defined by each recursive rule.

a) $T_n = T_{n-1} + 2n, T_1 = 0$

n	1	2	3	4	5	30
T_n	0	4	10	18	28	928 ✓

✓ ✓

b) $a_{n+1} = \frac{a_n}{n+1}$, $a_1 = 12$

n	1	2	3	4	5	10
a_{n+1}	12	6	2	$\frac{1}{2}$	$\frac{1}{10}$	3.3×10^{-6} ✓

✓ ✓

c) $U_n = U_{n-1} + U_{n-2}, U_0 = 1\ U_1 = 2$

n	1	2	3	4	5	12
U_n	2	3	5	8	13	377

✓ ✓ ✓

Question Two: [2, 4, 2, 2: 10 marks]

Angelena has built a new school. When the school first opens she has 230 students across the school with the population of the school set to increase by 24% each year.

a) Assuming no students leave the school, state the recursive rule to describe the number of students at the end of each year.

$$T_{n+1} = 1.24T_n \quad T_0 = 230$$

✓ ✓

For the first two years no students leave the school for other schools and no students are old enough to graduate yet. However, after the initial first two years, 50 students leave the school at the end of each year.

b) Calculate the number of students at the end of the 4th year assuming the population continues to grow at 24% each year.

$$T_2 = 354 \ (nearest\ whole)$$

$$T_{n+1} = 1.24T_n - 50 \quad T_0 = 354$$

✓ ✓ ✓

$$4th\ year\ is\ 2\ years\ later\ T_2 = 432.31$$ ✓

$$\approx 432\ students$$

c) Explain why Angelena would not be worried about losing 50 students per year but would be very concerned if she was losing 100 students per year.

✓ ✓

Currently the 24% growth rate is more than50 students. However, it is less than 100 so losing 100 would cause an overall decrease in her school population.

9 years after Angelena opened her new school, Brad opened an even better school very nearby and Angelena's students begin flocking to Brad's new school. Her student population no longer increases but reduces by 24% each year.

d) Describe what happens to Angelana's school over the next few years.

The falling number of students at Angela's schools follows a pattern of exponential decline. There is a significant drop in population numbers the first year after the Brad's school opens and then the population numbers begin to fall a lot slower. As the population gets smaller 24% becomes a less significant number. Her school population becomes very small within 10 years of Brad's school opening. ✓ ✓

Question Three: [2, 2, 2, 2: 8 marks]

Consider the following sequence:

Term	1	2	3	4	5	n
T_n	1	1+2	1+2+3	1+2+3+4	1+2+3+4+5	$\frac{n(n+1)}{2}$

1 3 6 10 15

a) Give the recurrence formula to calculate the n^{th} term.

$$T_n = T_{n-1} + n \quad T_1 = 1$$

b) Show that the n^{th} term can be calculated using the formula $\frac{n(n+1)}{2}$.

$$T_5 = 15 \quad T_5 = \frac{5(5+1)}{2} = \frac{30}{2} = 15$$

c) Calculate the 9th term and 10th term.

$$T_9 = 45 \quad T_{10} = 55$$

d) Complete the following table where S_n is the sum of successive terms.

n	1	2	3	4	5
S_n	1	4	10	20	35

Question Four: [2, 2, 2, 5: 11 marks]

Cole buys an antique piano for $2 100. The seller tells Cole that the piano is an investment as its value is certain to increase by 5% each year.

a) Given that Cole purchased the piano for current market value, what will the piano be valued at in 12 years time?

$$2\,100 \times 1.05^{12} = \$3771.30$$

b) How long will it take before the piano's value is at least $8 000?

$$2100 \times 1.05^{t} = 8000$$

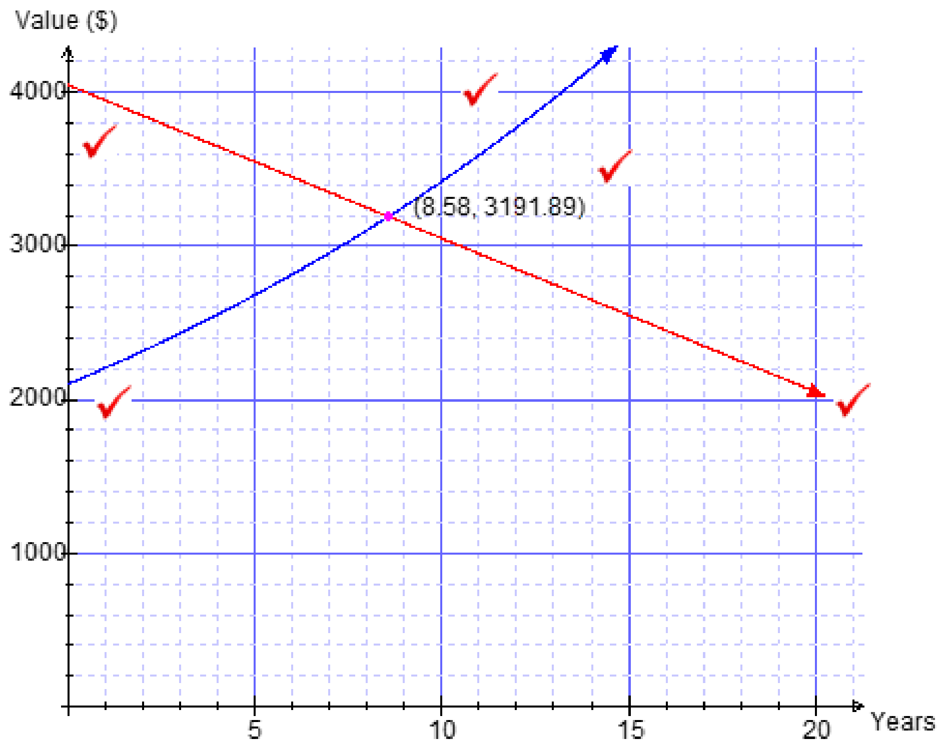
$$t = 27.4 \;\therefore\; after\ 28\ years.$$

Ty is doing some investing and buys $4 050 worth of shares into a relatively new company. Shortly after his purchase the company begins to run into some serious trouble and the value of Ty's investment falls by $100 each year.

b) Give the recurrence formula for the value of Ty's shares each year.

$$T_n = T_{n-1} - 100 \quad T_0 = 4050$$

c) Sketch the graph of the value of each man's purchase given the patterns described continue. Label the key features and the point at which the value of the piano and shares are equal.



Question Five: [3, 2, 2: 7 marks]

In an arithmetic sequence S_n is the sum of successive terms. Consider the arithmetic sequence where, $S_1 = 8, S_2 = 23, S_3 = 45, S_4 = 103$.

a) Calculate the first three terms of the sequence.

$$T_1 = 8 \quad ✓$$

$$T_2 = 23 - 8 = 15 \quad ✓$$

$$T_3 = 45 - 23 = 22 \quad ✓$$

b) Calculate the 18th term of the sequence.

$$T_n = 8 + 7(n - 1) \quad T_{18} = 127$$

✓ ✓

c) Calculate S_{12}.

$S_{12} = 558$ ✓ ✓